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Prescribing Effective Human Problem-Solving Processes: Problem Description in Physics

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We formulate a theoretical model specifying the underlying knowledge and procedures whereby human problem solvers can generate useful initial descriptions of scientific problems. This model is prescriptive, that is, it does not necessarily try to simulate the behavior of actual experts nor assume that their performance is optimal. To test such a model, formulated for the domain of mechanics, we devised a carefully controlled experiment where human subjects were induced to act in accordance with specified alternative models and where their resulting performance was observed in detail. The results show that the proposed model is sufficient to generate excellent problem descriptions, that these markedly improve subsequent problem solutions, and that major components of the model are necessary for good performance. Furthermore, detailed analysis of the data shows how the model predictably prevents the occurrence of many common errors. Such a validated prescriptive model provides a useful basis for teaching students improved problem-solving skills.

Problem solving is an intellectually demanding activity of central importance in any science. Hence it is a challenging task to design instructional methods for teaching students good scientific problem-solving skills. To be effective, such instructional design must be based on an adequate understanding of how good scientific problem-solving performance can be achieved and of how novice students perform before instruction.

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Recent years have witnessed considerable interest in analyzing cognitive processes and knowledge structures underlying problem-solving performance in several scientific domains, for example, in geometry (Greeno, 1978), physics (Simon & Simon, 1978), and computer programming (Polson & Jeffries, 1982). Problem solving in the domain of physics has been studied by an especially large variety of approaches. A number of investigations have examined the problem-solving performance of subjects of different levels of expertise (Chi, Feltovich, & Glaser, 1981; Larkin, McDermott, Simon, & Simon, 1980a; Larkin & Reif, 1979; Simon & Simon, 1978). Other studies have identified naive conceptions or misconceptions of physics students (Champagne, Klopfer, & Anderson, 1980; Clement, 1982; diSessa, 1982; McCloskey, Caramazza, & Green, 1980; Trowbridge & McDermott, 1980, 1981; Viennot, 1979). Efforts have also been made to develop process models of physics problem solving. These have included psychological models to describe and simulate the performance of human subjects of different levels of ability (Larkin, 1981; Larkin, McDermott, Simon, & Simon, 1980b), models recently summarized by Chi, Glaser, & Rees (1982). They have also included some artificial-intelligence models embodied in computer programs less concerned with simulating human performance (Bundy, 1978; Bundy, Byrd, Luger, Mellish, & Palmer, 1979; Byrd & Borning, 1980; de Kleer, 1977; Luger, 1981; Novak, 1977).

Although studies of the thought processes of experts and novices have yielded valuable insights about effective problem solving, they have significant limitations. For example, it is unwise to assume that the performance of experts is necessarily optimal. Furthermore, educational efforts cannot merely aim to teach students to perform as experts do. Instead, they must often teach students to use explicit procedures to accomplish tasks which experts perform almost automatically because of years of experience.

This paper describes our efforts to study human problem solving from a more general point of view which transcends the investigation of naturally occurring intellectual functioning. Our aim has been to specify cognitive processes and knowledge structures which lead to good human problem solving in a realistic scientific domain, *without* necessarily trying to simulate what actual experts do and *without* assuming that experts always perform optimally. Such a "prescriptive" approach is more general than a descriptive one since it allows greater freedom for theoretical inventiveness and deliberate experimentation. For example, although a prescriptive theoretical model of good intellectual performance may be partly suggested by naturalistic observations of experts, it may also be proposed on the basis of purely theoretical task analyses. Indeed, the sole criterion of validity of such a prescriptive model is that it lead to effective performance, even if it does not mimic what actual experts do.

An analogy may help to clarify the distinction between a prescriptive point of view and a descriptive one. Imagine that a hypothetical cognitive scientist, working in the era of Julius Caesar, had been trying to formulate a theoretical model of good performance in arithmetic problem solving. A prescriptive model of good performance might conceivably have proposed use of the (now familiar) place-value representation of numbers. This would have been an excellent *prescriptive* model since it would have led to very good human performance. But it would have been an unsatisfactory *descriptive* model since contemporary experts used Roman numerals.

A prescriptive approach is common in artificial intelligence. As pointed out elsewhere (Reif, 1979), it can also be of major interest in work on human cognitive processes. From a purely scientific point of view, prescriptive studies of human cognition have the following virtues: (1) By focusing attention on questions transcending naturalistic functioning, they can provide more general insights about the thought processes and forms of knowledge leading to effective human performance. (2) They can provide a basis of comparison useful for analyzing the performance of actual novices or experts, and can thus help reveal unsuspected tacit knowledge underlying expert behavior. (3) Finally, they provide theoretical insights about how to improve human performance beyond existing levels of expertise.

From the more applied point of view of "human knowledge engineering," prescriptive studies of human cognition are important in education since systematic approaches to instruction require explicit models of the thought processes whereby students are to achieve good final performance. Prescriptive studies are also essential for designing human-computer interaction since such designs must explicitly specify effective forms of human thought as well as effective forms of computer operation.

The following comments summarize the general approach adopted in our work to formulate and test a prescriptive model of human intellectual performance.

The *formulation* of such a prescriptive model involves the following major steps:

- *Specification of applicability:* Specify the conditions under which the proposed model is supposed to be applicable. These conditions include the characteristics of the tasks to be performed within some specified knowledge domain, and the characteristics (capabilities and limitations) of the persons who are to perform these tasks.

- *Formulation of a model of good performance:* Formulate a prescriptive model specifying explicitly the procedures and forms of knowledge whereby a person, with the previously specified characteristics, can perform effectively the specified kinds of tasks.

- *Formulation of alternative models:* Formulate alternative prescriptive models differing in specific ways from the model of good performance. (Comparison between such alternative models then allows one to ascertain which particular features of these models are necessary or sufficient for good performance.)

The subsequent *testing* of any such prescriptive model is based on the following paradigm: Induce persons to act in accordance with the model and observe whether their resulting performance has the predicted characteristics (e.g., that it is effective in the expected ways). The implementation of this paradigm involves the following major steps:

- *Elaboration of the model:* Elaborate the model into the form of a detailed implementable program explicating the procedures and associated factual knowledge specified by the model.

- *Measures to ensure implementability:* Test and modify this program to specify it at an optimal level of detail, so that all steps in this program are readily interpretable and reliably executed.

- *Controlled experiments:* Design and carry out controlled experiments in which individual persons are induced to act in accordance with the program elaborating the model. Make detailed observations of their resulting performance. (We shall discuss later the particular experimental methods we used to induce subjects to perform in specified ways.)

Although prescriptive models of human performance could also be tested by computer simulation, experimental tests with human subjects have the advantage of providing *direct* evidence of the validity of such models for their intended human use. Furthermore, experiments with human subjects can test models in a form directly suitable for the design of human instruction.

Note that the preceding approach for formulating and testing a prescriptive model of effective performance by humans is analogous to that used in artificial intelligence for models of effective performance by computers.

In the following pages we apply the prescriptive approach to one important aspect of scientific problem solving, namely the initial redescription of problems into a form which facilitates their subsequent solution. We first outline the formulation of a prescriptive model specifying how to generate useful initial problem descriptions in a particular scientific domain (exemplified by mechanics). Then we discuss in greater detail the elaboration of this model and of a modified model into testable forms, and the implementation of particular experimental methods to test these models with human subjects. Finally, we discuss the results obtained in these experiments and point out some of their educational implications.

PRESCRIPTIVE MODEL OF PROBLEM DESCRIPTION

Specification of Applicability

In our study of scientific problem solving, we focused on problems in basic college-level physics, specifically in the field of mechanics. Problem solving in this domain is realistically complex, representative of other quantitative scientific or engineering fields, and often difficult for many students. It is also of practical importance and a serious challenge in physics teaching. On the other hand, this domain is sufficiently simple and well-defined to be amenable to an analysis of underlying cognitive processes. Furthermore, such an analysis can draw upon insights derived from previous observations of experts and novices in this domain.

We presuppose that the human subjects engaging in such problem solving have not only typical human limitations (e.g., limitations of short-term memory), but also relatively complex human capabilities (e.g., the ability to understand English, to draw diagrams, to do algebra, and to interpret individual principles in basic physics). On the other hand, we do *not* assume that these subjects have the more sophisticated abilities needed to use physics principles in the integrated ways needed for solving nontrivial problems. Indeed, our interest is precisely in formulating prescriptive models specifying this more sophisticated knowledge.

Formulation of a Model of Effective Problem Description

Outline of a Prescriptive Model of Problem Solving

We have presented elsewhere a prescriptive theoretical model of effective human problem solving in the domain of physics (Reif & Heller, 1982). This model specifies some general procedures to be used in conjunction with a knowledge base about a particular scientific domain. The general procedures subdivide the problem-solving process into three major stages: (1) the generation of an initial problem description, and qualitative analysis, designed to facilitate the subsequent construction of a problem solution; (2) the generation of the actual solution by methods which facilitate the decision making required for efficient search; and (3) the assessment and improvement of this solution. The domain-specific knowledge base is designed to facilitate these general procedures. It contains declarative knowledge of concepts and principles, together with specific procedures facilitating their use, and is organized hierarchically to provide descriptions at various levels of detail.

In this paper we study in greater detail one component of this prescriptive model, namely the process whereby a human subject can generate a useful initial description (or "representation") of any problem. Although our study deals with only one aspect of this model of effective problem solving, this aspect is of crucial importance. Indeed, the initial description generated for a problem often determines how easily this problem can subsequently be solved or whether it can be solved at all. Yet, novice problem solvers commonly fail to describe problems adequately. Furthermore, observations of experts usually reveal rather little detailed information about their description processes since experts tend to redescribe problems rapidly, and almost automatically, on the basis of large amounts of tacit knowledge.

According to our prescriptive model, the generation of a useful initial problem description can conveniently be decomposed into two successive stages. In the first of these, a person starts from an originally presented problem and uses general *domain-independent* knowledge to generate a "basic description" of this problem. This basic description merely transforms the problem into a readily interpretable form. It summarizes explicitly the information specified and wanted in the problem, identifies relevant time-dependent processes and decomposes them into distinct subprocesses, introduces useful symbols, and expresses the relevant information in convenient symbolic representations (in pictorial as well as verbal forms). The generation of this basic description (discussed more fully in Reif & Heller, 1982) is not trivial; novice students commonly make mistakes even in this relatively simple part of the description process. However, to narrow the scope of the present paper, we shall restrict our attention to the second, more complex and interesting, stage of the description process, the generation of a "theoretical description" of a problem.

Model of Theoretical Problem Description

The knowledge base for any domain specifies the particular entities of interest in this domain, the special concepts useful for describing these entities, the special properties of these concepts, and principles and rules expressed in terms of these special concepts.

A "theoretical description" of a problem is a description deliberately expressed in terms of the special concepts and properties in the knowledge base. The generation of a theoretical description of a problem brings it into a form incorporating much important knowledge needed for its solution and simplifying the application of all relevant principles. The subsequent solution of the problem is, therefore, greatly facilitated.

The declarative knowledge in the knowledge base specifies implicitly the ingredients needed for the theoretical description of any problem in this domain. However, the actual generation of a theoretical description is greatly facilitated by the availability of an *explicit procedure* specifying how to describe any problem by the coherent application of domain-specific knowledge.

Our prescriptive model of problem description specifies, therefore, an explicit procedure for generating a theoretical description from a basic description of any problem in a specified domain. This procedure, which accompanies the knowledge base for this domain and exploits its declarative knowledge, thus specifies the following: (1) How to identify the particular entities which should be described in any particular problem. (2) How to apply special concepts to describe these entities. (3) How to exploit particular properties of these concepts. (4) How to apply particular principles in the knowledge base to check that the description is self-consistent and correct.

Theoretical Description in Mechanics

The preceding general remarks can be illustrated and made more specific in the case of *any* particular knowledge domain. For example, the knowledge base for the scientific domain of mechanics provides the following information:

It specifies that the particular entities of interest in mechanics are particles (objects sufficiently small or simple to be adequately described like geometric points) or systems consisting of many such particles (e.g., rigid bodies or strings).

The knowledge base introduces two kinds of special concepts to describe such particles. One kind, used to describe *individual* particles, includes concepts to describe the intrinsic characteristics of particles (e.g., "mass") and other concepts to describe the motion of particles (e.g., "position," "velocity," "acceleration"). The other kind, used to describe the *interaction* between particles, includes concepts such as "forces."

The knowledge base for mechanics also specifies important properties of these special concepts. For example, various "interaction laws" specify how concepts describing interaction are related to concepts describing motion (e.g., how the force exerted on one particle by another is related to the positions of these particles). Such interaction laws are specified for various kinds of interactions occurring in nature. Some of these interactions are "short-range", that is, they are only appreciable if the interacting particles are so close as to "touch" each other. By contrast, the other interactions are "long-range", that is, they can be appreciable even if the interacting particles are separated by some distance. (The most common example is the gravitational interaction of a particle with the earth.)

Finally, the knowledge base for mechanics includes important "motion principles" which specify how the motion of particles changes as a result of interactions between them. (For example, one such principle is Newton's famous "second law" $m\mathbf{a} = \mathbf{F}$ which relates the acceleration $\mathbf{a}$ of a particle to the force $\mathbf{F}$ exerted on it by all other particles.)

According to our prescriptive model, the generation of effective problem descriptions in mechanics can be ensured if the knowledge base for this domain includes an explicit description procedure. This procedure should specify how the declarative knowledge in mechanics can be applied coherently to generate a good theoretical description of any problem in this domain. This procedure should thus specify the following: (1) How to identify the particular entities (particles) of interest in any problem. (2) How to describe these particles by special concepts for describing motion ("position," "velocity," and "acceleration") and special concepts for describing interaction ("forces"). (3) How to exploit the special properties specified for these concepts by the various interaction laws. (4) How to check the correctness of the resulting description by ascertaining its consistency with certain motion principles.

Our prescriptive model leads thus to the formulation of the procedure, summarized in Table 1, for generating a theoretical description of any problem in mechanics. This procedure is elaborated more fully in later sections of

TABLE 1
Procedure for Generating a Theoretical Problem Description in Mechanics

Relevant times and systems: At each relevant time (previously identified in the basic description of the problem) identify those systems relevant in the problem because information about them is wanted, or because they interact with such systems directly or indirectly.

Description of relevant systems: At each relevant time, describe in the following way each relevant system (if simple enough to be considered a single particle), introducing convenient symbols and expressing simply related quantities in terms of the same symbol:

> *Description of motion:* Draw a "motion diagram" indicating available information about the position, velocity, and acceleration of the system.
>
> *Description of forces:* Draw a "force diagram" indicating available information about all external forces on the system. Identify these forces as follows:
>
> > *Short-range forces:* Identify each object which touches the given system and thus interacts with it by short-range interaction. For each such interaction, indicate on the diagram the corresponding force and all available information about it.
> >
> > *Long-range forces:* Identify all objects interacting with the given system by long-range interactions. (Ordinarily this is just the earth interacting by gravitational interaction.) For each such interaction, indicate on the diagram the corresponding force and all available information about it.

Checks of description: Check that the descriptions of motion and interaction are qualitatively consistent with known motion principles (e.g., that the acceleration of each particle has the same direction as the total force on it, as required by Newton's motion principle $m\mathbf{a} = \mathbf{F}$).

the paper and in Appendix A. Examples of theoretical descriptions generated by this procedure are shown in our later discussion of experimental results, as well as in Reif and Heller (1982).

Implications of the Model of Description

Important features of the model. Despite its seeming simplicity, the description procedure proposed by the model, as outlined in Table 1, is far from trivial. Application of the procedure ensures that highly important declarative knowledge in the knowledge base is systematically and correctly incorporated in the initial description of any problem. Such an initial description should greatly facilitate the solution of that problem.

The following kinds of declarative knowledge are explicitly incorporated in the description of any mechanics problem when the prescribed procedure is applied.

The relation between motion and interaction is central to the science of mechanics. Accordingly, the description procedure requires that *both* the motion *and* the interaction of any particle be carefully described. By contrast, most physics textbooks (e.g., Resnick & Halliday, 1977) emphasize the need to describe forces, but *not* the corresponding need to describe motion.

In the science of mechanics, the concept of "force" is introduced to describe the *interaction* between objects. Correspondingly, the description procedure requires that *interacting objects* should be identified *before* the specification of particular forces describing their interaction. Hence the procedure guards against errors caused by students' prescientific conceptions (Clement, 1982; Viennot, 1979) in which forces are viewed as ultimate causal agents, producing effects independently of the existence of other objects.

As mentioned previously, the science of mechanics includes the knowledge that interactions between objects can be classified into two types, short-range and long-range interactions. The description procedure uses this knowledge explicitly by enumerating all "touching objects" in order to identify all short-range forces. The procedure thus helps to ensure the complete enumeration of all relevant forces, without inadvertent omissions.

Finally, the science of mechanics includes important motion principles. The procedure exploits these deliberately to check the correctness of any generated problem description. (For example, the procedure includes checks of the consistency between the motion and interaction of any particle, and of the consistency between the forces acting on mutually interacting particles, as required by Newton's second and third laws.)

Predicted performance resulting from the model. Application of the description procedure, specified by our theoretical model in Table 1, is expected to have the following major consequences:

1. The description procedure should generate an explicit and detailed initial description of any mechanics problem in terms of the special concepts of this scientific domain. This description should be appreciably more explicit than descriptions overtly generated by actual experts or descriptions commonly presented in textbooks.

2. The description procedure should help subjects to avoid many of the errors commonly committed by students. For example, the explicitness of the procedure should help to avoid errors due either to the omission of relevant forces or to the enumeration of nonexistent forces.

3. The description procedure should sometimes lead to significantly easier reformulations of certain problems. For example, a question asking "when does a particular string become slack" would be translated by the procedure into the question "when does the force exerted by the string become zero." Such a reformulated question, involving the properties of familiar forces, is much more easily interpreted and answered.

4. The explicit problem description generated by the procedure of Table 1 should appreciably facilitate the subsequent solution of a problem. Indeed, this description incorporates already much of the relevant information needed for the selection and generation of equations.

Formulation of Alternative Models

The description procedure outlined in Table 1 is based on a prescriptive model (which we call model M) specifically designed to produce a "good" theoretical description of any mechanics problem, that is, an explicit description facilitating substantially the subsequent solution of the problem. For purposes of comparison, it is useful to formulate at least one modified model which lacks selected features of the proposed model of good performance. Experiments with such a modified model can then reveal whether the particular features omitted from the original model are actually necessary for good performance.

Accordingly, we also formulated a modified model (which we call model M*) which is appreciably less explicit than model M. This model M* specifies a description procedure chosen to be similar to the advice which physics textbooks (e.g., Resnick & Halliday, 1977) commonly provide about the description of mechanics problems. This description procedure differs from that specified by the original model in Table 1 in these major respects: (1) It does *not* include an explicit description of the motion of a system. (2) It specifies that a separate force diagram should be drawn for each system and that this diagram should indicate *all* forces exerted on this system by other systems. However, it provides no explicit directions about *how* to identify all these forces or to exploit their known properties. (3) It does not systematically check the resulting description's consistency with available physics principles.

Like model M, this modified model M* is elaborated more fully in a later section and is exhibited in detail in Appendix A.

TESTING THE MODEL OF PROBLEM DESCRIPTION

As mentioned in the introduction, our basic paradigm for testing a prescriptive model of human performance is to induce subjects to act in accordance with the model, and to observe whether their resulting performance is effective in the predicted ways. The implementation of this paradigm requires specially controlled conditions to ensure that a human subject acts in a prescribed manner. We now outline our general approach for implementing this paradigm and then describe the details of our experiment.

Approach Used to Test a Prescriptive Model

Our experimental approach is to create conditions where a human subject is induced to act in specified ways by being placed under "external control." To clarify this approach by an analogy, consider the familiar situation where a pilot lands his or her plane in bad weather while following directions from an air-traffic controller on the ground. Under these conditions, a human information processor (the pilot) makes extensive use of his or her sophisticated knowledge, but relegates higher level control of this knowledge to external directions. This situation can be viewed as an experiment with the following interesting characteristics: (1) It allows a separation of high-level control knowledge from lower level implementation knowledge. For example, if the plane were to crash, the information retrievable from the taped conversation between the pilot and ground-control would allow one to distinguish whether the crash occurred as a result of appropriate control directions improperly implemented by the pilot, or whether it occurred as a result of faulty control directions. By contrast, if a pilot crashed the plane while flying entirely under his or her own control, one could *not* distinguish whether the fault was in the pilot's higher level control knowledge or in lower level implementation knowledge. (2) A set of control directions, specifying how to land a plane, can be viewed as a theory specifying how a human subject, with sophisticated human capabilities, can land a plane. In other words, such control directions would constitute a good theory of plane landing if, and only if, the correct execution of these directions leads to reliably effective landings. (3) Such a validated theory could ultimately be used as the basis of a theory of *instruction* for landing planes. Such a theory would need to teach human subjects to internalize, and carry out independently, the control directions which had previously been external.

Let us now turn from this analogy to external-control experiments designed to test prescriptive models of human performance, specifically, models of effective problem description. To carry out such experiments, one needs first to design a detailed "program," consisting of step-by-step directions and associated factual knowledge, whereby a human subject can be guided to act in accordance with a specified model of performance. For example, such a program might guide a human subject explicitly to execute the description procedure outlined in Table 1. The program should be problem independent, leading to a good description of *any* problem in the specified domain. (This is *unlike* the analogy of plane landing where ground-control directions are usually situation specific.) The individual directions must be appropriately matched to the characteristics and preexisting knowledge of the intended human subjects, that is, they must be detailed enough to be reliably interpretable and executable, but not so detailed as to be distracting.

In the experimental procedure an individual subject is asked to carry out specified tasks (here, the description and subsequent solution of various problems) by executing directions that are successively read to the subject according to the program specified by the model. The subject is asked to talk about his or her thought processes and the session is tape-recorded. Detailed data can thus be gathered about the subject's written output and verbalized thought processes generated while responding to the external-control directions.

Such detailed observations allow one to obtain the following kinds of information to test the proposed model of performance:

1. One can ascertain whether the proposed model of good performance is *sufficient* to lead to good performance. This can be done by determining whether subjects, working under external control in accordance with the model, do indeed achieve good performance. (Such experiments do not necessarily imply that the proposed model is unique since other models might conceivably lead to equally good or better performance.)
2. One can verify that the prerequisite basic knowledge, which the model presupposes of human subjects, is by itself *not* sufficient to produce good performance. This can be done by letting subjects, with such knowledge, work *without* external guidance of the model and observing that their performance is then poor.
3. One can ascertain whether selected features of the proposed model are, in fact, *necessary* to achieve good performance. This can be done by comparative experiments where human subjects work under external control of a modified model which lacks selected features of the proposed model of good performance. Predictable performance deficiencies should then occur.
4. Finally, one can test whether the proposed model of good performance leads to specific predicted features in the resulting performance. For example, one can ascertain whether, and how, the occurrence of specific errors is prevented when human subjects act in accordance with the model.

It should be emphasized that the aim of such external-control experiments is to ascertain the merits of a proposed model of good performance, but *not* to teach. Subjects may, of course, learn incidentally while working under conditions of external control. However, such learning need not occur, because external control directions may not become internalized. For example, a subject, performing very well while working under external control, might revert to poor performance if that control were to be removed.

The next three sections discuss the application of the preceding approach to test the proposed model of problem description. In particular, we first discuss the elaboration of the proposed model M, and of the modified model M*, into detailed programs. Then we describe preliminary studies designed to test and modify these programs to ensure that they are reliably implementable by human subjects. Finally, we discuss the actual experiment which compared the problem-solving performance of three different groups of subjects: a group M guided by external-control directions based on the proposed model M, a group M* guided by similar directions based on the modified model M*, and a "comparison" group C working without any external guidance.

Elaboration of the Models

The full model M and the modified model M* each specify problem-description procedures to be used in conjunction with relevant factual knowledge about the problem domain of mechanics. The procedures in each model were elaborated into detailed external-control directions. The relevant factual knowledge, which is the same for both models, was summarized in written form so that subjects could refer to it during problem-solving sessions.

The external-control directions for generating problem descriptions were supplemented with some additional directions to guide subjects' subsequent solutions of the described problems. These directions provided minimal guidance for generating and combining equations. They were essentially the same for both groups M and M*, and will not be discussed further.

The elaboration of a prescriptive model of human performance (such as the description procedure outlined in Table 1) into implementable control directions requires careful attention to the following important issues:

1. Each direction must be readily *interpretable and executable* by human subjects possessing the knowledge and other characteristics presupposed by the model. (For example, a direction to "draw an interaction diagram" would only be interpretable if a subject knows what an interaction diagram is; it would only be executable if the subject has adequate knowledge about the forces involved in the interaction.) Making directions interpretable involves refining the wordings of directions for adequate clarity, providing examples of desired performance (e.g., sample interaction diagrams), and expressing directions at an appropriate level of detail. The level of detail must be suffi-

cient to make a direction clearly interpretable, but not so excessive as to be distracting or disruptive. Ideally, the level of detail should be adjusted to the knowledge and capabilities of the subject for whom the direction is intended, that is, a direction should spell out only that knowledge which is not already reliably available to the subject.

2. Every step must be *actually executed* and in the *proper sequence.* Therefore, the directions must be sufficiently explicit, and provide enough control, to ensure that all needed activities are performed.

3. Directions must be executed *correctly and completely.* Hence various checks need to be included to verify that directions have been implemented properly.

As discussed and exemplified later, adequate pretesting is crucial for refining directions until they meet the preceding criteria.

External-Control Directions for the Model M

Table 1 summarizes the procedure specified by our model M for generating effective descriptions of problems in mechanics. The elaboration of this model into detailed step-by-step directions is shown in Appendix A. This elaboration involved translating the procedure of Table 1 into specific directions that could be easily implemented by human subjects familiar with the relevant basic concepts of mechanics. The following activities were required to achieve our specified criteria of implementability.

To achieve interpretability, the steps in the description procedure were first expressed as easily comprehensible and natural-sounding directions. The steps in Table 1 thus yielded corresponding directions to construct separate motion and interaction descriptions for each relevant system (steps 3, 4, 7, and 11 in Appendix A) and directions to check that these descriptions are qualitatively consistent with known mechanics principles (steps 17, 18, and 25 in Appendix A). Similarly, comments about symbolism in Table 1 were translated into specific reminders to choose convenient symbols to represent the values of quantities in a problem (steps 5 and 8).

Additional steps had to be introduced to ensure that steps would be executed in proper sequence. For example, the procedure of Table 1 prescribes that motion and interaction descriptions be constructed for each system, one system at a time, before the solution of a problem is attempted. Correspondingly, directions in Appendix A include specific steps to coordinate these activities in cycles of choosing a particular system (steps 1 and 2), describing the motion and interaction of this system, determining whether any other systems remain to be described (step 19), and branching accordingly (step 20).

Further steps were needed to provide adequate control over subjects' use of the declarative knowledge available to them. Pretesting revealed that subjects, even when prompted, would sometimes fail to use factual knowledge readily available in the summary provided to them. Accordingly, the directions included explicit mention of some especially important elements of declarative knowledge, specifically, knowledge about properties of the acceleration of systems (step 6 in the Appendix) and properties of certain forces on systems (steps 9 and 10). These steps ensure that a general procedure is used in conjunction with appropriate declarative knowledge, for example, that the procedure for describing forces is coupled with domain-specific knowledge about the types of forces exerted by various kinds of systems.

Finally, several additional steps were added as checks to ensure that previous steps had been performed completely and correctly. Such checks are necessary since human subjects tend to be fallible and distractable, prone to forget steps in a procedure or to disregard available information. Accordingly, the elaborated directions in Appendix A contain explicit checks that all interacting systems have been considered (steps 12 to 16), that all available information in the problem statement has been considered (step 24), and that symbols have been conveniently chosen (steps 21 to 23). These checks are in addition to the more general checks mentioned in Table 1 (corresponding to steps 17, 18, and 25 in Appendix A).

*External-Control Directions for the Modified Model M**

As mentioned previously, the modified model M* was introduced for purposes of comparison with the proposed description model outlined in Table 1. The modified model, designed to simulate somewhat the descriptive advice commonly found in physics textbooks, is thus considerably less complete and explicit than the proposed model M. The detailed differences between the elaborated versions of model M and model M* are exhibited in Appendix A. The main differences between these versions are the following: (1) The full model M includes descriptions of both motion and interaction for every system, while Model M* includes only a description of interactions. (2) The model M includes a detailed procedure specifying *how* to enumerate all forces on a system, while model M* includes only a direction to enumerate all forces on the system by other objects. (3) The full model M contains some explicit references to elements in the knowledge base (to some particular properties of motion and interaction), while such explicit references are omitted in model M*. (4) The full model M includes some powerful checks based on general physics principles, while the modified model M* does not include such checks.

Summary of Relevant Factual Knowledge

In addition to the external-control directions, we prepared a printed summary of basic mechanics principles required for the solution of problems used in our study. This summary included information about the properties of relevant motion descriptors (e.g., equations specifying the components of a particle's acceleration for circular motion), information about the properties of various kinds of interactions (e.g., magnitudes and directions of forces), and information about relevant motion principles (e.g., Newton's law of motion $m\mathbf{a} = \mathbf{F}$). Subjects in all three experimental groups could refer to this summary at any time.

Measures to Ensure Implementability

Since testing of the prescriptive model depends heavily on the comparison of performance guided by external-control directions, it is extremely important that these directions are implementable by the subjects (i.e., that they are readily interpretable, actually executed, and implemented correctly). To ensure such implementability, the directions were pretested with pilot subjects, and practice activities were designed to familiarize experimental subjects with the directions.

Tests and Modification of the Control Directions

The external-control directions evolved to their final state over a 6-month period of pilot testing and revision. The aim was to phrase the directions in such a way that they communicated clearly what a subject was expected to do. As mentioned before, the directions had to be specific enough to provide sufficient guidance, but not so detailed that they became more confusing than helpful. While these considerations are difficult to operationalize, they made themselves manifestly evident during pretesting. It was quite apparent when subjects had no idea what they were supposed to do, or grew impatient or bewildered because directions were too minutely detailed.

In some respects, the process of developing the directions was like writing a computer program: The desired performance simply would not result until the sequence of steps was adequate for the task. We began with a set of directions that represented our best hypotheses about the ways to express the procedures and facts in the model. These directions were then tried out with pilot subjects, using problems of a kind similar to those used in the actual experiments. Each such trial revealed unclear directions, missing or redundant steps, or illogical sequences—and suggested appropriate modifications to correct such deficiencies. After several trials, we finally arrived at directions which guided subjects through problem descriptions and solutions without serious mishaps.

Many of the modifications were required because of subjects' inability to understand particular steps in the directions. For example, the check of consistency between motion and interaction (steps 17 and 18 in Appendix A) was a particularly difficult procedure to express clearly and succinctly. Several versions were tried out before the current wording was selected for use. Our difficulty in developing these particular directions revealed the amount of tacit knowledge required for comparing motion and interaction descriptions. It was not sufficient simply to direct subjects to make sure that motion and interaction descriptions were consistent with a specified principle. In order for the direction to be interpretable and executable, it was necessary to indicate *how* to determine whether they were consistent. Earlier, less specific, versions of these steps were met with requests for clarification.

The need for other changes was revealed by subjects' repeated errors when responding to our directions. Description errors that commonly occurred during pilot testing included omission of normal or friction forces, incorrect determination of the direction of normal or friction forces, and incomplete determination of the acceleration of particles in circular motion. These errors occurred despite the fact that the relevant information was available to subjects in the summary given to them. Hence it became apparent that control over the actual use of this information would have to be explicitly assumed by the external-control directions. The inclusion of directions with appropriate references to the knowledge base (steps 6, 9, and 10 in Appendix A) ensured that the relevant information was actually considered, and eliminated the errors previously observed in the pilot testing.

Familiarizing Subjects with Control Directions

Practice directions. To familiarize subjects with the control directions, we developed M and M* practice directions incorporating selected excerpts from the full M or M* control directions. Subjects were given pretraining by following these practice directions to describe a simple problem situation.

Sample solution. To supplement the external-control directions, each subject in groups M and M* was provided with a printed sample solution of a simple problem. This sample solution included examples of the motion diagrams and force diagrams mentioned in the directions. Some directions could then refer to this sample solution (e.g., "Draw a motion diagram like the one in the sample solution..."). The sample solution was described and introduced to the subjects during the initial practice with the directions. Subjects working under external control could look at this sample solution at any time during the experimental sessions.

Experimental Method

Problem Tasks for Assessing Performance

Three approximately matching pairs of mechanics problems (listed in Appendix B) were selected from commonly used introductory physics texts

(French, 1971; Resnick & Halliday, 1977; Symon, 1971) and reworded slightly for increased clarity.

All of the problems used in the study could be solved by application of one fundamental motion principle, Newton's second law (**ma** = **F**). Two of the three pairs of problems (1A, 1B; 3A, 3B) required nontrivial force descriptions because they involved several forces (both long-range and short-range). These problems were included to allow assessment of procedures for enumerating forces. The third pair of problems (2A and 2B) required nontrivial motion descriptions; they involved systems in circular motion, the analysis of which is frequently performed incorrectly by novices. These problems were included to allow assessment of procedures for describing motion.

Our main concern was with the theoretical description of problems, rather than with the prior basic description designed to make a problem readily comprehensible to a problem solver. Hence we supplemented the verbal statement of each problem with a basic description (of the form explicitly indicated in the case of problem 1A in Appendix B) consisting of a diagram of the problem situation, a list of all information specified in the problem, and a symbolic statement of the problem goal.

The pairs of problems were split into two approximately equivalent sets, A and B. Half of the subjects in each group received one set as a pretest and the other set during the experimental treatment sessions; the other half of the subjects received these sets in the opposite order.

Subjects

The subjects in the experiment were 24 paid volunteers, all undergraduate students currently enrolled in the second course of an introductory physics sequence at the University of California at Berkeley. In the first course of this sequence the subjects had studied the physics principles of mechanics and had, in homework assignments and course examinations, solved mechanics problems of the kinds used in our experiments. Furthermore, the subjects were randomly selected from those volunteers who had received a grade of B – or better in their previous physics course. Therefore the subjects used in our study could be assumed to have acquired recently, and with at least nominal competence, a basic knowledge of the physics principles relevant to the problems used in our study.

These subjects were randomly assigned to the three experimental groups, eight subjects to each group.

Procedure

General conditions. Problems were individually administered to each subject in all sessions. All subjects solved one set of problems in a pretest session while working without external guidance. Before solving the second set of problems, subjects in groups M and M* were introduced to the experimental procedure of working under external control. These subjects then solved the problems under the guidance of directions read to them by the experimenter, while subjects in group C worked without such guidance. Subjects were asked to talk aloud about what they were thinking while solving the problems, and their verbalized statements were recorded with their permission. All subjects, during all sessions, had access to the printed summary of mechanics principles and to the sample solution previously given to them.

Because our interest was not in the subjects' knowledge about algebra or trigonometry, apparent errors in the application of such knowledge were pointed out or corrected by the experimenter when they occurred.

Introduction to the study. The first session began with a discussion of the purpose of the research. The subjects were told that we were concerned about the difficulty students have solving physics problems, and that we wished to observe them solving such problems. Our stated aim was to identify the methods used by students and to understand their difficulties. The subjects were then told that they would be solving moderately difficult mechanics problems, and were given detailed instructions about the experimental procedures that would be used.

We then provided the subjects with the printed summary of relevant mechanics principles, a summary to which they were encouraged to refer at any time. They were informed that every principle needed for the problems was in this summary, and were asked to read through the summary for as much time as they wanted before working on the problems. The subjects were also told that the only motion principle listed in the summary, and required for the solution of these problems, was Newton's second law $m\mathbf{a} = \mathbf{F}$ (i.e., that the problems did not require use of the energy or momentum principles, nor of principles relevant to rotational motion). Our motivation for providing this information was to let students concentrate on the problem description processes of interest to us, without their being distracted by demands to remember various bits of factual information or to decide between various motion principles.

Pretest. A pretest, consisting of three mechanics problems, was administered individually to each subject working without external guidance. A subject ordinarily required between 45 minutes to an hour to solve these three problems.

Many subjects expressed considerable uncertainty while solving these problems. They often stated that they had no idea how to begin working, that they did not know what to do next once they had begun, and that they were convinced there was no systematic way to solve these problems. In response to subjects' direct questions and requests for help, the experimenter urged the subjects to proceed as if the problems were part of a regular homework or test. The subjects were also told that any of their questions would be answered at the end of their final session.

Practice. After completing the pretest problems, the experimental procedure was explained to the subjects in groups M and M*. The subjects were told that we had some problem-solving methods we wished to test by observing what happened when subjects used them. The subjects were then given practice in following selected steps of the directions (using the practice directions described previously). The verbal directions read to the subjects were supplemented by the printed sample solution, already described, to which subjects could refer at any time during the practice or treatment sessions. This practice period lasted between 20 and 30 minutes.

Treatment. About 1 week after the pretest (and practice session), each subject returned for one or two subsequent sessions to solve three more problems. Subjects in groups M and M* were guided through the solution of these problems, while those in the comparison group C worked again without external guidance.

Subjects working with external guidance were read the standard directions from a script, one step at a time. Each direction had to be performed by the subject before the next one was read. As long as the subject implemented the direction, the experimenter considered that direction executed, regardless of whether it had been done correctly. (For example, during construction of a force diagram, subjects were directed to indicate all forces exerted on a system by other objects. If a subject described such a force in the wrong direction, the step was nevertheless considered executed. As another example, subjects in group M were asked whether any object, other than those already named, touched the system of interest. Even if the subject responded in the negative when there was another touching object, the step was nevertheless considered executed.) However, if instead of answering the question or performing the step, the subject prematurely skipped to a later step, he or she was stopped and the direction was repeated until an appropriate response was made.

The methodology of external control was initially not always readily accepted by the subjects. A frequently encountered difficulty was the subjects' resistance to surrendering control to the experimenter. Many seemed determined to solve the problems in their own way, rather than to follow the directions given to them. They were then reminded that their solutions during the first session had already provided us with information about their own methods, and that we now needed their help to assess *our* methods. For the most part, this reminder was sufficient to enlist the subjects' cooperation.

Once their initial resistance had been overcome, many subjects became overtly positive in their response to the directions. Several remarked, with notable surprise, that these steps "really work" and that the problems seemed suddenly "easy" to solve.

Furthermore, the directions were very easily implemented by the subjects. Indeed, after working through the first problem, several subjects afterwards took over control at times, guiding themselves through the solutions according to our procedure without waiting for the experimenter's directions. For example, they would say, "Okay, now the interaction diagram. Block A touches, and it exerts normal and friction forces...and long-range is the earth, gravity." At these times the experimenter would let the subjects both state the next step and do it, and would intervene only when a step was missed. (The ease with which subjects did internalize some of the steps is an encouraging indication of the learnability of the procedures, although this experiment was *not* designed to teach.)

Data Analysis

In order to assess the quality of subjects' problem-solving behavior, it was necessary to identify and define performance measures. Table 2 summarizes the criteria used as measures of good performance and the major classes of errors used to assess deficiencies in performance.

TABLE 2
Performance Measures and Error Types

Performance Measure	Major Error Types
Adequacy of motion information: Was information about the magnitude and direction of each system's acceleration correctly included in the equations?	Wrong direction of acceleration. Wrong magnitude of acceleration.
Adequacy of interaction information: Were all required forces included in the equations? Were directions and magnitudes of those forces correctly indicated?	Missing force. Wrong direction of a force. Wrong magnitude of a force.
Adequacy of equations: Were the number and kinds of equations generated sufficient to determine a solution? Were all equations correctly instantiated?	Missing required equation. Incorrect information contained in equation. Meaningless equation (e.g., inconsistent choices of systems).
Correctness of final answer: Was correct answer obtained?	Incorrect (or no) final answer.

The first two measures in Table 2 refer to the adequacy of motion and interaction information used in the *solution* of a problem. This adequacy was assessed by examining the equations generated by the subjects and ascertaining whether correct information about acceleration and forces was included in these equations. Thus this assessment considered only implicit descriptions (reflected in the equations), whether or not explicit descriptions were presented in diagrams. In this way we avoided giving unfair advantage to subjects, in groups M and M*, who were directed to draw diagrams.

The third measure in Table 2, the adequacy of equations, was assessed by determining both whether the solution contained a sufficient number of equations, and whether all of the individual equations were correct. Solutions were not penalized if unneeded equations were generated by the subjects.

Finally, solutions were assessed with respect to the correctness of the final answer.

All assessments of solutions were made by inspection of the subjects' written work, with reference to their verbal protocols as needed. In general, the adequacy of equations and correctness of the final answer could be judged easily from the subjects' written work. However, consideration of verbal protocol data was sometimes required to assess the extent to which equations reflected adequate descriptions of motion or interaction. For example, if a subject wrote an equation containing only symbols for forces, but no term involving the acceleration, the subject's verbal statement could help establish whether the subject deliberately determined the acceleration to be equal to zero, or whether the subject simply ignored the motion and blithely assumed that "forces balance." The subjects' recorded comments, while solving the problems, usually cleared up any such questions of interpretation. If a clear judgment could not be made, even with the aid of the protocol data, the subject was given the benefit of the doubt and the information was considered correct.

Results

The adequacy of every solution was assessed with respect to the performance measures listed in Table 2. The data summarized in Table 3 and Figure 1 show the mean number of each student's solutions (on the three problems solved during pretest or treatment sessions) that were correct on each of these measures. The data are summarized for students in each of the three treatment groups, M, M*, and C. The right-most columns in Table 3 indicate which of the differences between these groups are statistically significant. Table 3 and Figure 1 also summarize the performance of all 24 students on the pretest. There were no significant differences between the various groups on

TABLE 3
Mean Number of Solutions with Correct Performance on Specified Measures

Performance Measures	Pretest[a]	Treatment[b] M	M*	C	Statistical Differences[c] M>M*	M*>C	M>C
Correct motion information	1.83	3.00	2.63	1.63			*
Correct force information	1.33	3.00	2.00	1.38	**		**
Sufficient and correct equations	0.83	2.88	1.63	0.75	**		**
Correct final answer	0.79	2.75	1.38	0.63	*		**

Note: Maximum score = 3.00.
[a] $n = 24$.
[b] $n = 8$ per group.
[c] Kruskal-Wallis Test results: $^{*}p<0.01$; $^{**}p<0.005$.

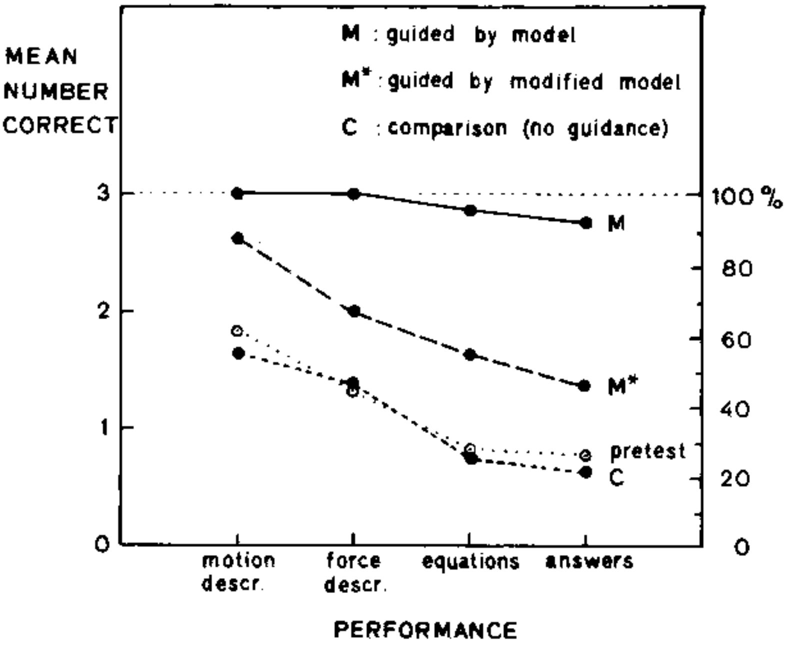

Figure 1. Graphs of the mean number of solutions (out of three) with correct performance on specified measures.

this pretest, nor between these pretest results and the performance of the comparison group C in the treatment.

Sufficiency of the Model

The purpose of this research was to evaluate selected aspects of the proposed model of good problem-solving performance in mechanics. The major question of interest is whether the kinds of procedures proposed by the model are sufficient for producing successful solutions. If the kinds of knowledge included in the model are sufficient, students working in accordance with the model would be expected to perform well.

The performance of subjects in group M, working under external control, indicates that the proposed procedures did indeed lead to very good performance. As is apparent from Table 3 and Figure 1, these students performed nearly perfectly: All of their solutions contained every required equation, and all of their equations contained correct and complete information about motion and interaction. (The slightly lower incidence of correct final answers resulted from incorrect combination of equations in problem 2B. Instead of performing a required vector addition, some students treated vectors like numbers.)

Inadequacy of Performance Unguided by the Model

As indicated in Table 3 and Figure 1, the subjects' performance on the pretest, and the performance of the comparison group C, indicate that the subjects' prior knowledge was definitely *not* sufficient to solve these kinds of problems adequately. On the average, the subjects solved correctly less than one third of the pretest problems. Furthermore, less than one third of their solutions contained enough equations to achieve a solution, and less than one half of these solutions incorporated correct information about both the motion and interaction of the relevant systems.

These results indicate that the kind of knowledge acquired by students as a result of ordinary instruction in an introductory mechanics course is not sufficient to endow them with the ability to solve fairly standard mechanics problems at the level of this course. Although the subjects *did* have enough basic knowledge of physics concepts and principles to interpret and implement the external-control directions used in our experiments, the additional procedural and factual knowledge provided by these directions was necessary to help the subjects achieve good problem-solving performance.

Necessity of Components of the Model

The results already discussed show that subjects, working in accordance with the proposed model, perform very well on problem-solving tasks. However, one may ask whether all components of this model are actually necessary.

This question can be partially answered by comparing the performance of group M, which worked in accordance with the proposed model, with the performance of group M*. As mentioned previously, if the knowledge components omitted from M* were in fact necessary for good performance, the observed performance of group M* should be less adequate than that of group M. In particular, since the differences between the models lay in the completeness and explicitness of procedures for constructing initial problem descriptions, the descriptions of motion and interaction by group M* would be expected to be inferior to those by group M. Correspondingly, the subsequent equations generated by subjects in group M*, and hence also the final problem answers obtained by them, should be less often correct than those generated by subjects in group M.

The data in Table 3 and Figure 1 reveal essentially this pattern of results. All results are statistically significant, except in the case of motion description, where the performance of group M* was not significantly poorer than the perfect performance of group M. It thus appears that at least some of the components, deleted from model M to create the modified model M*, are indeed necessary for achieving good problem solutions.

Detailed Analysis of Subjects' Performance

A closer examination of the subjects' performance provides insights into the way in which the proposed model facilitates good performance. In particular, the following paragraphs discuss the effects of particular components of the model and how they prevent the occurrence of errors commonly committed by students working without guidance by the model.

Procedure for Enumerating Forces

One of the most common errors committed by subjects working without external control was the omission of one or more relevant forces acting on a system. About 75% of the subjects omitted some relevant forces in at least one of their pretest problem solutions.

These difficulties in problem description may be illustrated by problem 3B. This problem (illustrated in Fig. 2 and described more fully in Appendix B) deals with two blocks, A and B, connected by a string passing over a fixed pulley. The block B is free to slide, with friction, relative to the horizontal floor beneath it and relative to the block A on top of it. The goal of the problem is to find magnitude of the force $\mathbf{F}_0$ needed to pull the block B to the left with constant velocity.

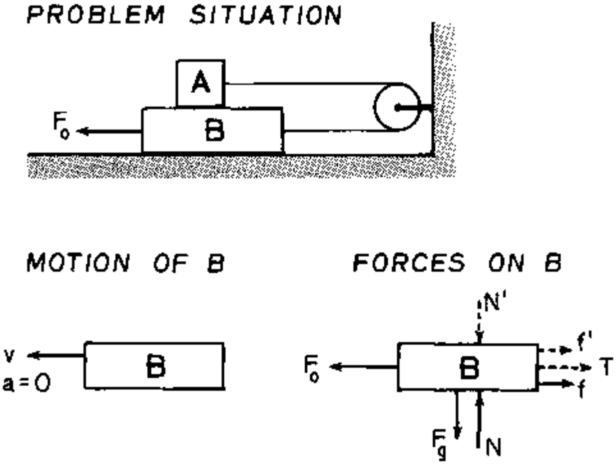

Figure 2. Problem 3B involving two blocks connected by a string, with motion and force descriptions of block B. (Forces frequently omitted by subjects are indicated by dashed arrows.)

When the description procedure of the model is applied to block B, it first generates a motion diagram which describes the velocity **v** and acceleration **a** of block B, as shown in Figure 2. Then the procedure generates an interaction diagram indicating all forces on block B. To do this, the procedure first identifies all objects which touch block B and all of the corresponding short-range forces exerted on B by these objects. As indicated in Figure 2, these forces are the applied force $\mathbf{F}_0$ exerted by the system pulling the block B, the tension force **T** exerted by the string, the normal and friction forces **N** and **f** exerted by the floor, and the normal and friction forces **N′** and **f′** exerted by block A. Then the procedure identifies the long-range gravitational force $\mathbf{F}_g$ exerted on block B by the earth.

Identification of all these forces presented particular difficulties for the subjects unguided by the model. For example, the friction force **f** on B by block A was omitted in half of the pretest solutions of this problem. The normal force **N′** on B by block A, and the tension force **T** on B by the string, were omitted in 25% of the pretest solutions.

By contrast, none of the subjects in group M omitted any forces in any of their solutions. One reason for this result is that the model's procedure for enumerating forces relies on the identification of all objects which touch the system of interest—and the identification of objects touching a given system is trivial for human subjects. Furthermore, the procedure includes an explicit reminder of factual knowledge in the knowledge base, that is, that the force exerted by a surface consists ordinarily of two component forces (the normal and friction forces) perpendicular and parallel to the surface. Hence the explicit description procedure eliminates the common error of omitting some short-range forces acting on a system.

In general, the subjects implemented this systematic procedure fairly easily once they became familiar with it. However, their comments did reflect the fact that they were quite unused to thinking of forces as being exerted *on* a particular system *by* other identifiable interacting systems. As one subject remarked, "I always get these As on Cs and Cs on As mixed up!" Furthermore, every subject in group M responded to the request for the name of a *system* by giving instead the name of a *force.* For example, a subject would say "gravity" instead of "the earth," or "tension" instead of "the string." Hence the direction to name a system had to be repeated frequently to ensure that it was indeed performed.

In this particular problem, subjects guided by the modified model M* also did not omit any forces on block B. (Apparently, the mere direction to indicate *all* forces exerted on block B by all the other systems, and later to check that there were no other forces on B by anything else, was in this case sufficient to lead to better performance than that exhibited by subjects working without any external guidance.) However, the subjects guided by the modified model M* *did* omit relevant forces in their solutions of all the *other* problems (as indicated in Table 3 and Fig. 1). By contrast, subjects guided by the full model M *never* omitted any relevant forces in any of the problems. Thus the detailed description procedure, specified by the full model, is significantly more reliable than the less explicit procedure specified by the modified model M*.

Special References to the Knowledge Base

A second very common error, exhibited on the pretest, was that of ascribing the wrong direction to a force. Half of the subjects made this error in at least one pretest solution. An example of this difficulty occurs in problem 1A (illustrated in Fig. 3 and described more fully in Appendix B). In this problem blocks A and B are connected by a thin light string which passes over a fixed pulley. It is specified that the block C, which lies on top of block A, remains at rest relative to A without sliding off.

To solve this problem, all the forces on block C must be correctly identified and described. However, in 83% of the pretest solutions of this problem, subjects asserted that the friction force on C by A was directed to the *left*, although it is actually directed to the right. (Indeed, if the friction force were directed to the left, block C would certainly slide off block A. It is only the friction force, exerted on C by A, which moves C to the right along with A.)

Verbal protocol data reveal that such errors, describing friction forces, result from subjects' use of an incomplete rule. In all the solutions, all but 2 of the 24 subjects in this study (i.e., 92% of them) made statements like "friction

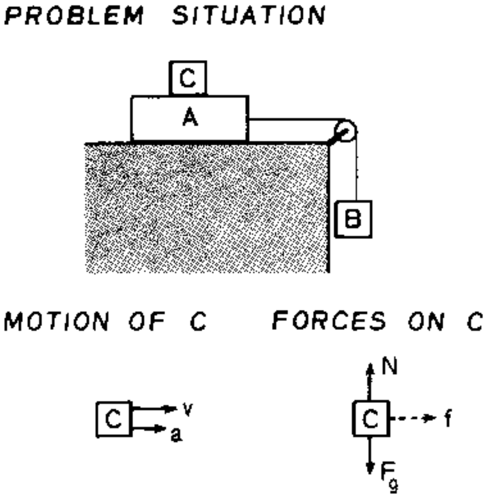

Figure 3. Problem IA involving three blocks, with motion and force descriptions of block C. (The friction force **f**, indicated by a dashed arrow, is frequently ascribed the wrong direction, i.e., to the left.)

tends to oppose the direction of motion" or "C is moving to the right, so friction would be to the left." These statements contain no explicit consideration of the *reference frame* relative to which motion is being described. Implicitly they seem to assume that only motion relative to the earth is relevant. Correspondingly, they also claim that the direction of a friction force is always opposite to the direction of motion relative to the earth.

This assumed characterization of the subjects' reasoning about friction forces seems to account quite well for their errors. Since the motion of block C in Figure 4 is to the right (relative to the earth), the subjects conclude that the friction force **f** on C is oppositely directed and thus is exerted to the left. Subjects in groups M* and C made this error as frequently as all subjects on the pretest problems.

The subjects' lack of attention to reference frames leads them to a deficient rule characterizing the properties of the friction force, a rule valid only in special circumstances. The correct rule, explicitly available to all subjects in the summary of factual knowledge given to them, states that the friction force has a direction opposing the *relative* motion of the interacting objects. In model M, the control directions explicitly refer to this information in the knowledge base (i.e., they ensure that subjects actually use the knowledge available to them). As a result, *all* subjects guided by the model M *correctly* identified the directions of friction forces. In particular, in the problem illustrated in Figure 3, the correct rule implies that the friction force on C by A has a direction opposing the motion of C *relative* to A. If there were no friction force on C in Figure 3, C would simply remain at rest relative to the earth while A moves to the right relative to the earth. Thus C would move to the left *relative to A*. To oppose this motion, the friction force on C must, therefore, be directed to the right.

Check for Consistency Between Motion and Interaction

The model not only provides explicit rules for correctly describing forces, but also includes checks to ensure that forces have been described properly. One such check requires that the descriptions of the motion and interaction of each system be qualitatively consistent with the fundamental motion principle $m\mathbf{a} = \mathbf{F}$, that is, that the acceleration of a particle have the same direction as the total force on it. In order to perform this check, both the motion and interaction of each system must have been described explicitly, as required by the model. Some examples of such descriptions are illustrated in Figures 2 and 3.

The power of this checking procedure can be illustrated in the case of the problem in Figure 3. It is quite easy for subjects to determine that the acceleration of block C is directed to the *right*. If such a subject then claims that the friction force on this block is directed to the *left*, the checking procedure would immediately reveal that the direction of this force is inconsistent with that of the acceleration and must therefore be incorrect. Thus this check provides a reliable method for detecting and correcting the common error of incorrectly ascribing the wrong direction to the friction force in this problem.

The explicit qualitative comparison of motion and force diagrams also seemed to provide subjects with a powerful graphic demonstration of the meaning of Newton's motion principle $m\mathbf{a} = \mathbf{F}$. Several subjects in group M spontaneously reacted to this comparison with comments like "Oh! I never thought about it that way before!" This kind of qualitative comparison procedure may have a substantial potential for enhancing students' understanding of physics principles, a potential which may be worthy of further investigation.

Check for Consistency Between Mutual Forces

Another check on the initial theoretical description of a problem involves determining whether mutual forces between interacting particles have been correctly described as equal in magnitude and opposite in direction, in accordance with Newton's third law. The problem illustrated in Figure 3 is again useful for demonstrating the utility of this check. In this problem situation, blocks A and C interact with one another, each exerting both normal and friction forces on the other. Thus, the friction force on A by C should be equal in magnitude, but opposite in direction, to the friction force on C by A.

As discussed previously, many subjects erroneously described the direction of the friction force on C by A as directed toward the left. By the same deficient reasoning (as well as by application of the correct rule), the friction force on A by C was properly described as directed to the left. These two conclusions are incompatible because the two friction forces must be oppositely directed. The check of mutual forces would thus reveal that an error has been made and that more careful analysis of the situation is called for.

This check would reveal an error whenever two mutual forces are not oppositely directed (indicating that the direction of at least one force has been described incorrectly), or when one of a pair of mutual forces has been omitted. In fact, such situations occurred very frequently in the observed solutions. For example, six of the eight subjects in group M initially described incorrectly forces whose mutual forces were described correctly, and discovered those errors during the later check of mutual forces. Similarly, six of the eight subjects in group M* omitted, or incorrectly described, forces whose mutual forces were correctly described. However, since these subjects were not directed to check the description of mutual forces, they did *not* discover or eliminate these errors.

DISCUSSION AND IMPLICATIONS

The work discussed in this paper has aimed to formulate and validate a prescriptive theoretical model specifying some of the knowledge and procedures leading to good human problem solving in a quantitative science such as physics. We have focused particular attention on the generation of effective initial problem descriptions which facilitate the subsequent solutions of such problems. We sought to specify explicit procedures for generating a theoretical problem description which deliberately redescribes any situation in terms of the special concepts specified by the knowledge base for the relevant scientific domain. In the science of mechanics these procedures specify explicitly how to describe the motion of any system in terms of concepts such as velocity and acceleration, how to describe the interaction of any such system in terms of specified kinds of forces, how to exploit special knowledge about the properties of such forces, and how to check the resulting description by its consistency with known physics principles.

Our results show that human subjects, induced to follow such description procedures under carefully controlled conditions, do indeed reliably generate explicit and correct descriptions of the motion and interaction of systems in mechanics problems. Furthermore, these descriptions markedly facilitate the subsequent construction of correct problem solutions.

The generation of effective initial problem descriptions is far from trivial. Indeed, our experiments show that many students, after receiving good grades in a recent course where they received formal instruction in mechanics, nevertheless generate incomplete and/or incorrect descriptions of fairly routine problems—and thus fail to solve them properly.

As we have pointed out, these problem-solving deficiencies exist even if students understand basic physics concepts and principles. They still lack the more strategic kinds of knowledge specified in our prescriptive model, that is, the meta-knowledge that it is important to describe a problem with care before attempting to search for its solution, explicit knowledge about what types of information should be included in an effective description, and explicit systematic procedures specifying how to generate such a description. These kinds of knowledge are usually possessed by experts, although predominantly in tacit form, and are rarely taught explicitly in physics courses. The work discussed here shows that such knowledge can be made more explicit and that, if used by students, it can strikingly improve their problem-solving performance.

Our theoretical ideas about the generation of effective initial problem descriptions have been illustrated in the particular scientific domain of mechanics. However, they can readily be extended to other scientific domains (e.g., to electric circuits, or thermodynamics, or even to domains outside of physics), provided that they are used in conjunction with the particular knowledge base of the relevant scientific domain.

The generation of effective initial problem descriptions is very important to achieve effective problem solving, but is not sufficient. A complete prescriptive theoretical model of effective problem solving must also deal with other central issues, for example, with decision processes facilitating the efficient search for a solution, with useful forms of organization of the knowledge base, and so forth. We have outlined such a more encompassing problem-solving model elsewhere (Reif & Heller, 1982) and hope to validate other aspects of this model by experimental methods similar to those used in our study of problem description.

Our experimental methods have involved the detailed observation of human subjects working under external control in accordance with prescriptive models of performance (either a proposed model of effective performance or an alternative model). This method permits one to explore in detail the efficacy of any proposed model of human task performance and to manipulate experimentally various parameters of such a model. Accordingly, this method may be broadly useful to study theoretical models specifying cognitive processes and knowledge structures for achieving intellectual performance in a wide variety of domains.

The work discussed in this paper is highly relevant to the design of instruction for teaching students improved scientific problem-solving skills. Such instruction requires a well-validated prescriptive model specifying how good problem solving is to be achieved by students as a result of instruction. (As pointed out in the introductory paragraphs, such a model must do more than merely simulate the problem-solving behavior of actual experts.) Our model for generating effective problem descriptions, together with the experiments verifying its efficacy, is thus an essential prerequisite for teaching students important problem-description skills needed for good problem solving.

Such teaching efforts would require students to internalize, and learn to use habitually, control knowledge which was explicitly externalized in our experiments. In other words, instructional design must use insights about good performance and then deal explicitly with the processes whereby such performance can be learned. Our model of problem description has already been quite useful in some of our practical efforts to teach problem-solving skills to students in physics courses. We hope to go beyond such informal efforts to develop more explicit and systematic instructional methods based on our analysis of relevant cognitive processes.

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APPENDIX A

EXTERNAL-CONTROL DIRECTIONS FOR PROBLEM DESCRIPTION

The following are the detailed external-control directions used in our experiment to elaborate the proposed description model M and modified model M*. Alternative directions used for model M or M* are marked by the letters M or M*, respectively. Directions marked "M only" occur only for model M, but are omitted in model M*. Directions not specifically marked are common to both models. The letters E and S refer to actions by the experimenter or subject, respectively, with statements made by either put between quotes.

Theoretical Description of Systems

M [E: "Let's now draw diagrams describing each system of interest."

M* [E: "Let's now draw diagrams describing the forces on each system of interest."

Choice of Particular System

E: "Which system . . .do you wish to consider (first)/(next)?" (1)

S: Names a system *'X'*.

If X is a string or is not affected by interactions with other systems:

> E: "There is no need to describe *X*." (2)
> Return to step 1.

Else continue:

M only

Motion Description

E: "First draw a motion diagram of *X*, including any available information about its position, velocity, and acceleration relative to a convenient reference frame. If the velocity or acceleration is zero, indicate that on your diagram." (3)

S: Draws motion diagram of *X*.

E: "It is also useful to include on this diagram any known properties of the system, such as mass." (4)

If previous systems have been described:

> E: "Be sure to use convenient symbols and to relate them to those you've used previously." (5)

If X has circular motion:

> E: "Remember, the acceleration of a system in circular motion ordinarily, although not always, has two components, one tangential and the other toward the center of the circle. Check to be sure whether both components exist in this case." (6)

Interaction Description

E: "Now let's draw an interaction diagram for *X*, using the method I've suggested."

Short-Range Forces

M

> E: "First name each system that touches *X*, including those that exert applied forces. (7)

As you identify each system, indicate all external contact forces exerted on X by that system."

M* E: "Draw a force diagram indicating the forces exerted on X by all other systems." (7′)

If previous systems have been described:

E: "Be sure to use convenient symbols and to relate them to those you've used previously." (8)

S: Names interacting systems ('Y') and/or indicates forces.

M only *If* interaction with surface:

E: "Remember, the force exerted by a surface ordinarily, although not always, has two components, the normal force and friction force. Check to be sure whether both components exist in this case. (9)

The normal force is perpendicular to the surface and directed away from it. The friction force opposes the *relative* motion of the contact points—here it opposes the motion of X relative to Y." (10)

Long-Range Forces

E: "Name all external systems that directly interact with X without touching it or through any other physical contact. Then indicate the long-range forces exerted on X by each such system." (11)

S: Names system and/or indicates force.

Check: Missing or Extraneous Forces

M E: "Are there any other systems touching X?" (12)

M* E: "Are there any other forces on X by anything else? (12′)

S: "Yes" or "no".

If yes:

M — E: "Draw the forces exerted by that (those) system(s)." (13)

M* — E: "Draw the forces." (13′)
Return to step 12.

M only — *Else* continue:

E: "Are there any other systems directly interacting with X by long-range forces?" (14)

S: "Yes" or "no".

If yes:

E: "Draw the force exerted by that system." (15)
Return to step 14.

Else continue:

E: "If not, you are finished describing all forces on X. Do not add any others. (16)

Check: Consistency Between Motion and Interaction

E: "The motion and interaction of the system must be consistent. In your diagrams, are the forces on X such that, with proper magnitudes, their vector sum can have the same direction as X's acceleration? Show me how you determine this. (You might want to check whether this is true by comparing components along convenient directions.)" (17)

S: Checks consistency; responds "yes" or "no" with explanation. Modifies description(s) if necessary.

E: "What would have to be true about the relative magnitudes of the forces on X for the acceleration and resultant force to have the same direction?" (18)

S: Describes required relative magnitudes of forces.

Repetition of Description for Each System

E: "Have all systems of interest been described yet?" (19)

S: "Yes" or "no".

If no:

Repeat theoretical description procedure, beginning at step 1. (20)

Else continue:

Check of Entire Description

E: "After describing all systems, it's useful to double-check your work. Let's run through a checklist to make sure you haven't missed anything."

Check: Choice of Useful Symbols

E: "All arrows should be labeled." (21)

S: Checks arrows.

E: "Except for the gravitational force (which may be expressed as "mg"), or any magnitudes actually given in the problem statement, the values of quantities should *not* be evaluated at this time. Symbols like "F," "T," and "N," with subscripts, should be used instead." (22)

S: Checks symbols.

E: "Look at the symbols in all of your diagrams. Wherever different symbols have been used, the values of these quantities should *actually* be unrelated. If values are the same or simple multiples, use the same symbol. If values are unrelated, different symbols should be used." (23)

S: Checks symbols.

Check: Use of all Information in Problem

E: "All information specified in the problem should be incorporated in your analysis. Please reread the problem carefully to make sure you have considered all the given information. In particular, make sure you've obtained from the problem all available information about the magnitude and direction of the velocity and acceleration of each system." (24)

S: Rereads problem statement. Modifies descriptions if needed.

Check: Exploitation of Constraints (Mutual Forces)

M only

E: "Check to make sure that all action-reaction pairs of forces are described as equal in magnitude and opposite in direction. For example, if systems *A* and *B* interact, the force of *A* on *B* in your diagram of *B* should be opposite in direction but should have the same magnitude as the force of *B* on *A* in your diagram of *A*. Look for forces between each pair of systems and check that they are described right." (25)

S: Checks forces.

APPENDIX B

PROBLEMS USED IN THE EXPERIMENT

Each of the problems included a basic description summary, shown here only in the case of problem 1A.

Problem 1A

Figure 4 shows a cart A (of mass 2m) free to move without friction along a horizontal table. This cart is attached by a light string, which passes over a pulley of negligible mass and negligible friction, to a block B (of mass m_B) suspended from the other end of the string. A block C (of mass m) lies on top of cart A. The coefficient of static friction between A and C is μ. What is the maximum value of m_B for which block C will remain on the cart without sliding?

Basic description summary (accompanying Fig. 4)

- Specified information:
  cart A: mass 2m;
  coefficient of static friction between A and C = μ.
  block B: mass m_B.
  block C: mass m;
  does not slide off A;
  coefficient of static friction between A and C = μ.
  string: massless.
  pulley: massless; frictionless.
  table: horizontal; frictionless.
- Goal:
  maximum m_B = ?

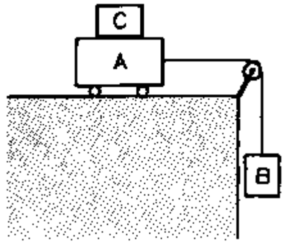

Figure 4. Diagram for problem 1A.

Problem 1B

Figure 5 shows a cart A, of mass m_A, which moves with negligible friction along a horizontal floor when it is pushed to the right by an applied force of magnitude F_0. A small block B, of mass m_B, is in contact with the right vertical side of the cart. The coefficient of static friction between the block and the side of the cart has a value μ. How large must be the magnitude F_0 of the applied force so that the block remains at rest relative to the cart, without slipping down?

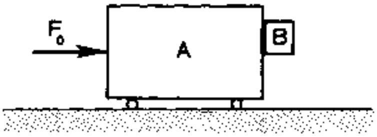

Figure 5. Diagram for problem 1B.

Problem 2A

A pendulum bob, of mass m, swings in a vertical plane at the end of a string of negligible mass fastened to the ceiling. At the highest point of its swing, the pendulum is in the position shown in Figure 6, with the string at an angle θ from the vertical. What is the magnitude of the tension force exerted on the bob by the string at this instant?

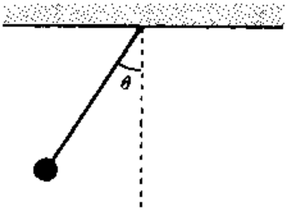

Figure 6. Diagram for problem 2A.

Problem 2B

An object, of mass m, slides along a circular track with negligible friction. When the object passes the point P in Figure 7, the magnitude of the force exerted on the object by the track is $3mg/\sqrt{2}$. What is the magnitude of the object's acceleration at that instant? (Use the values: $\sin 45° = \cos 45° = 1/\sqrt{2}$.)

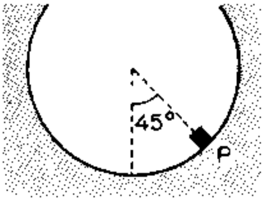

Figure 7. Diagram for problem 2B.

Problem 3A

A man, of mass m, stands on a board, of mass M, which he previously placed on a mud-covered hilly surface making an angle θ with the horizontal. The man holds on to a rope (of negligible mass and parallel to the surface of the hill) whose other end is fastened to a wall at the top of the hill. (See Fig. 8.) The man finds, to his dismay, that the board beneath him starts sliding down the hill. The coefficient of sliding friction between the man's shoes and the board is μ_1, and the coefficient of sliding friction between the board and the surface of the hill is μ_2. What is the magnitude of the acceleration $\mathbf{a}_B$ with which the board beneath the man slides down the hill while the man, holding on to the rope, remains at rest relative to the ground?

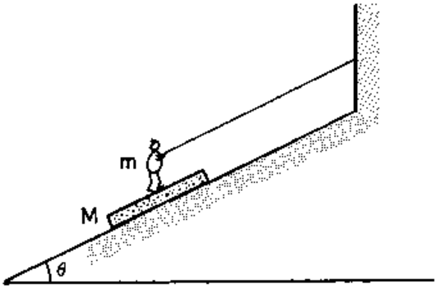

Figure 8. Diagram for problem 3A.

Problem 3B

Two blocks A and B are connected by a light flexible string passing around a frictionless pulley of negligible mass. (See Fig. 9.) Block A has a mass m_A and block B has a mass m_B. The coefficient of sliding friction between the two blocks, and also between block B and the horizontal floor below it, has a value μ. What is the magnitude F_0 of the force necessary to pull block B to the left at constant speed?

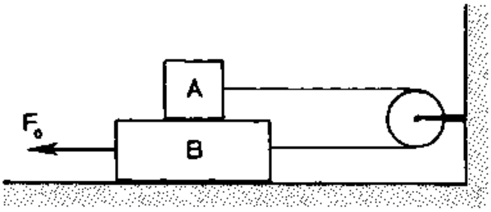

Figure 9. Diagram for problem 3B.

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